1. Introduction

It was recognized by Flamm [1] in (1916) that our universe may not be simply connected, there may exist handles or tunnels now called wormholes, in the spacetime topology linking widely separated regions of our universe or even connected us with different universes altogether. traversable Lorentzian wormholes "which have no horizons, allowing two-way passage through them", and were especially stimulated by the pioneering works of Morris, Thorne and Yurtsever [2, 3], where static, spherically symmetric Lorentzian wormholes were defined and considered to be an exciting possibility for constructing time machine models with these exotic objects, for backward time travel [4, 5, 6].

Morris and Thorne (MT) wormholes are static and spherically symmetric and connect asymptotically flat spacetimes. The metric of this wormhole is given by

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \,, \tag{1}$$

where $\Phi(r)$ being the redshift function and $b(r)$ is the shape function. The shape function describes the spatial shape of the wormhole when viewed. The metric (1) is spherically symmetric and static. The coordinate r is nonmonotonic in that it decreases from $+\infty$ to a minimum value b_0, representing the location of the throat of the wormhole, and then it increases from b_0 to $+\infty$. This behavior of the radial coordinate reflects the fact that the wormhole connects two separate external universes. At the throat $r = b = b_0$, there is a coordinate singularity where the metric coefficient g_{rr} becomes divergent but the radial proper distance

$$l(r) = \pm \int_{b_0}^{r} \frac{dr}{\sqrt{1 - b(r)/r}}, \tag{2}$$

must be required to be finite everywhere [7]. At the throat, $l(r) = 0$, while $l(r) < 0$ on the left side of the throat and $l(r) > 0$ on the right side. For a wormhole to be traversable it must have no horizon which implies that g_{tt} must never allowed to be vanish, i.e., $\Phi(r)$ must be finite everywhere.

Traversable Lorentzian have been in vogue ever since Morris, Thorn and Yurtsever [3] came up with the exciting possibility of constructing time machine models with these exotic objects. (MT) paper demonstrated that the matter required to support such spacetimes necessarily violates the null energy condition. Semiclassical calculations based on techniques of quantum fields in curved spacetime, as well as an old theorem of Epstein et. al. [8], raised hopes about generation of such spacetimes through quantum stresses.

There have been innumerable attempts at solving the "exotic matter problem" in wormhole physics in the last few years [5, 9]. Alternative theories of gravity [10] evolving wormhole spacetimes [11]$\sim$ [14] with varying definitions of the throat have been tried out as possible avenues of resolution.

Møller modified general relativity by constructing a new field theory in Weitzenböck spacetime [15]. The aim of this theory was to overcome the problem of energy-momentum complex that appears in the Riemannian spacetime [16]. The field equations in this new theory were derived from a Lagrangian which is not invariant under local tetrad rotation. Sáez [17] generalized Møller theory into a scalar tetrad theory of gravitation. Meyer [18] showed that Møller theory is a special

The tetrad theory of gravitation based on the geometry of absolute parallelism [21]~[28] can be considered as the closest alternative to general relativity, and it has a number of attractive features both from the geometrical and physical viewpoints. Absolute parallelism is naturally formulated by gauging spacetime translations and underlain by the Weitzenböck spacetime, which is characterized by the metric condition and by the vanishing of the curvature tensor using the affine connection resulting from the geometry of the Weitzenböck spacetime. Translations are closely related to the group of general coordinate transformations which underlies general relativity. Therefore, the energy-momentum tensor represents the matter source in the field equation for the gravitational field just like in general relativity.

It is the aim of the present work to derive a spherically symmetric solutions in the tetrad theory of gravitation. To do so we first begin with a tetrad having spherical symmetry with three unknown functions of the radial coordinate [29]. Applying this tetrad to the field equations of Møller's theory we obtain a set of non linear partial differential equations. We propose a specific restriction on the form of the stress-energy that when solving the non linear partial differential equations, automatically leads to a class of wormhole solutions. The characterization of our class of self-dual wormholes is

$$\rho = \rho_t = 0, \tag{3}$$

where ρ and ρ_t are respectively the energy density measured by a static observer and the convergence density felt by a timelike congruence. Indeed Eq. (3) characterizes a class of "self-dual" wormhole spacetimes which contains the Schwarzschild solution. The notation of duality involves the interchange of the active and passive electric parts of the Riemann tensor (termed as electrogravity duality) [13]. Electrogravity duality essentially implies the interchange of the Ricci and Einstein tensors. For vanishing Ricci scalar tensor the Ricci and Einstein tensors become equal and the corresponding solution could be called "self-dual" in this sense.

The weak energy condition says that the energy density of any system at any point of spacetime for any timelike observer is positive (in the frame of the matter this amounts to $\rho > 0$ and $\rho+p \geq 0$). When the observer moves at the speed of light it has a well defined limit, called the null energy condition (in the frame of matter $\rho + p \geq 0$) [30].

Under the duality transformation, ρ and ρ_t are interchanged indicating invariance of Eq. (3). Since energy densities vanish and yet the spacetime is not entirely empty, the matter distribution would naturally have to be exotic (violating the energy condition) [13]. Physically, the existence of such spacetimes might be doubted because of this violation of the weak and null energy conditions. Analogous to the spatial-Schwarzschild wormhole, for which $g_{00} = -1$ and $g_{11} = (1 - \frac{2m}{r})^{-1}$, these spacetimes have zero energy density but nonzero pressures [13]. The spatial-Schwarzschild wormhole is one specific particular solution of the equations $\rho = 0$, $\rho_t = 0$. The central problem of the traversable wormhole is connected with the unavoidable violation of the null energy condition. This means that the matter which should be a source of this object has to possess some exotic properties. For this reason the traversable wormhole cannot be represented as a self-consistence solution of Einstein's equations with the usual classical matter as a source because the usual matter is sure to satisfy all the energy conditions [31]. Therefore, it is a basic fact for the construction of the traversable wormholes that the null energy condition has to be violated [30].

It is our aim to discuss the physical properties of the spherically symmetric wormholes obtained. In section 2 a brief survey of Møller's tetrad theory of gravitation is presented. The exact solutions of the set of non linear partial differential equations are given in section 3. In section 4 the energy content of these solutions are given. Discussion and conclusion of the obtained results are given in section 5.

2. Møller's tetrad theory of gravitation

In a spacetime with absolute parallelism the parallel vector fields $e_i{}^\mu$ define the nonsymmetric affine connection

$$\Gamma^\lambda{}_{\mu\nu} \stackrel{\text{def.}}{=} e_i{}^\lambda e^i{}_{\mu,\,\nu}, \tag{4}$$

where $e^i{}_{\mu,\,\nu} = \partial_\nu e^i{}_\mu$. The curvature tensor defined by $\Gamma^\lambda{}_{\mu\nu}$ is identically vanishing, however.

Møller's constructed a gravitational theory based on this spacetime. In this theory the field variables are the 16 tetrad components $e_i{}^\mu$, from which the metric tensor is derived by

$$g^{\mu\nu} \stackrel{\text{def.}}{=} \eta^{ij} e_i{}^\mu e_j{}^\nu, \tag{5}$$

where η^{ij} is the Minkowski metric $\eta_{ij} = \text{diag}(+1\,,-1\,,-1\,,-1)$.

We note that, associated with any tetrad field $e_i{}^\mu$ there is a metric field defined uniquely by (5), while a given metric $g^{\mu\nu}$ does not determine the tetrad field completely; for any local Lorentz transformation of the tetrads $b_i{}^\mu$ leads to a new set of tetrads which also satisfy (5). The Lagrangian L is an invariant constructed from $\gamma_{\mu\nu\rho}$ and $g^{\mu\nu}$, where $\gamma_{\mu\nu\rho}$ is the contorsion tensor given by

$$\gamma_{\mu\nu\rho} \stackrel{\text{def.}}{=} \eta^{ij} e_{i\,\mu} e_{j\nu;\,\rho}, \tag{6}$$

where the semicolon denotes covariant differentiation with respect to Christoffel symbols. The most general Lagrangian density invariant under the parity operation is given by the form [15]

$$\mathcal{L} \stackrel{\text{def.}}{=} \sqrt{-g}\,(\alpha_1 \Phi^\mu \Phi_\mu + \alpha_2 \gamma^{\mu\nu\rho}\gamma_{\mu\nu\rho} + \alpha_3 \gamma^{\mu\nu\rho}\gamma_{\rho\nu\mu})\,, \tag{7}$$

where

$$g \stackrel{\text{def.}}{=} \det(g_{\mu\nu}), \tag{8}$$

and Φ_μ is the basic vector field defined by

$$\Phi_\mu \stackrel{\text{def.}}{=} \gamma^\rho{}_{\mu\rho}. \tag{9}$$

Here α_1, α_2, and α_3 are constants determined by Møller such that the theory coincides with general relativity in the weak fields:

$$\alpha_1 = -\frac{1}{\kappa}, \qquad \alpha_2 = \frac{\lambda}{\kappa}, \qquad \alpha_3 = \frac{1}{\kappa}(1 - 2\lambda), \tag{10}$$

where κ is the Einstein constant and λ is a free dimensionless parameter*. The same choice of the parameters was also obtained by Hayashi and Nakano [22].

Møller applied the action principle to the Lagrangian density (7) and obtained the field equation in the form

$$G_{\mu\nu} + H_{\mu\nu} = -\kappa T_{\mu\nu}, \qquad K_{\mu\nu} = 0, \tag{11}$$

where the Einstein tensor $G_{\mu\nu}$ is the Einstein tensor defined by

$$G_{\mu\nu} \stackrel{\text{def.}}{=} R_{\mu\nu}(\{\}) - \frac{1}{2} g_{\mu\nu} R(\{\}),$$

where $R_{\mu\nu}(\{\})$ and $R(\{\})$ are the Ricci tensor and Ricci scalar. $H_{\mu\nu}$ and $K_{\mu\nu}$ are given by

$$H_{\mu\nu} \overset{\text{def.}}{=} \lambda \left[\gamma_{\rho\sigma\mu}\gamma^{\rho\sigma}{}_{\nu} + \gamma_{\rho\sigma\mu}\gamma_{\nu}{}^{\rho\sigma} + \gamma_{\rho\sigma\nu}\gamma_{\mu}{}^{\rho\sigma} + g_{\mu\nu}\left(\gamma_{\rho\sigma\tau}\gamma^{\tau\sigma\rho} - \frac{1}{2}\gamma_{\rho\sigma\tau}\gamma^{\rho\sigma\tau} \right) \right], \tag{12}$$

and

$$K_{\mu\nu} \overset{\text{def.}}{=} \lambda \left[\Phi_{\mu,\nu} - \Phi_{\nu,\mu} - \Phi_{\rho}\left(\gamma^{\rho}{}_{\mu\nu} - \gamma^{\rho}{}_{\nu\mu} \right) + \gamma_{\mu\nu}{}^{\rho}{}_{;\rho} \right], \tag{13}$$

and they are symmetric and skew symmetric tensors, respectively.

Møller assumed that the energy-momentum tensor of matter fields is symmetric. In the Hayashi-Nakano theory, however, the energy-momentum tensor of spin-1/2 fundamental particles has non-vanishing antisymmetric part arising from the effects due to intrinsic spin, and the right-hand side of antisymmetric field equation (11) does not vanish when we take into account the possible effects of intrinsic spin.

It can be shown [23] that the tensors, $H_{\mu\nu}$ and $K_{\mu\nu}$, consist of only those terms which are linear or quadratic in the axial-vector part of the torsion tensor, a_{μ}, defined by

$$a_{\mu} \overset{\text{def.}}{=} \frac{1}{3}\epsilon_{\mu\nu\rho\sigma}\gamma^{\nu\rho\sigma}, \qquad where \qquad \epsilon_{\mu\nu\rho\sigma} \overset{\text{def.}}{=} \sqrt{-g}\delta_{\mu\nu\rho\sigma}, \tag{14}$$

where $\delta_{\mu\nu\rho\sigma}$ being completely antisymmetric and normalized as $\delta_{0123} = -1$. Therefore, both $H_{\mu\nu}$ and $F_{\mu\nu}$ vanish if the a_{μ} is vanishing. In other words, when the a_{μ} is found to vanish from the antisymmetric part of the field equations, (11), the symmetric part of Eq. (11) coincides with the Einstein field equation in teleparallel equivalent of general relativity.

3. Spherically Symmetric Solutions

Let us begin with the tetrad [29] (lines are labelled by l and columns by μ)

$$(e_l{}^{\mu}) = \begin{pmatrix} A & Dr & 0 & 0 \\ 0 & B\sin\theta\cos\phi & \dfrac{B}{r}\cos\theta\cos\phi & -\dfrac{B\sin\phi}{r\sin\theta} \\ 0 & B\sin\theta\sin\phi & \dfrac{B}{r}\cos\theta\sin\phi & \dfrac{B\cos\phi}{r\sin\theta} \\ 0 & B\cos\theta & -\dfrac{B}{r}\sin\theta & 0 \end{pmatrix}, \tag{15}$$

where A, D, B, are functions of the redial coordinate r. The associated metric of the tetrad (14) has the form

$$ds^2 = -\frac{B^2 - D^2 r^2}{A^2 B^2}dt^2 - 2\frac{Dr}{AB^2}drdt + \frac{1}{B^2}dr^2 + \frac{r^2}{B^2}(d\theta^2 + \sin^2\theta d\phi^2). \tag{16}$$

As is clear from (16) that there is a cross term which can be eliminated by performing the coordinate transformation [32]

$$\frac{ADr}{} \tag{17}$$

using the transformation (17) in the tetrad (15) we obtain

$$(e_l{}^\mu) = \begin{pmatrix} \dfrac{\mathcal{A}}{1-\mathcal{D}^2R^2} & (R\mathcal{D}-R^2\mathcal{D}\mathcal{B}') & 0 & 0 \\[2ex] \dfrac{\mathcal{A}\mathcal{D}R\sin\theta\cos\phi}{1-\mathcal{D}^2R^2} & (1-R\mathcal{B}')\sin\theta\cos\phi & \dfrac{\cos\theta\cos\phi}{R} & -\dfrac{\sin\phi}{R\sin\theta} \\[2ex] \dfrac{\mathcal{A}\mathcal{D}R\sin\theta\sin\phi}{1-\mathcal{D}^2R^2} & (1-R\mathcal{B}')\sin\theta\sin\phi & \dfrac{\cos\theta\sin\phi}{R} & \dfrac{\cos\phi}{R\sin\theta} \\[2ex] \dfrac{\mathcal{A}\mathcal{D}R\cos\theta}{1-\mathcal{D}^2R^2} & (1-R\mathcal{B}')\cos\theta & -\dfrac{\sin\theta}{R} & 0 \end{pmatrix}, \qquad (18)$$

where $\mathcal{A}$, $\mathcal{D}$ and $\mathcal{B}$ are now unknown functions of the new radial coordinates R which is connected to the old redial coordinate r through the transformation defined by

$$R = \frac{r}{B}, \qquad with \qquad \mathcal{B}' = \frac{d\mathcal{B}(R)}{dR}. \qquad (19)$$

We are interested in finding some special solutions to the partial non linear differential equations resulting from applying the tetrad (18) to the field equations (11).

<u>The First Solution</u>

If the unknown function $\mathcal{D}(\mathcal{R}) = 0$, then the resulting partial non linear differential equations have the form

$$\rho(R) = -\frac{1}{R\kappa}\left\{3\,\mathcal{B}'^2R - 2\,R\mathcal{B}''(R) + 2\,R^2\mathcal{B}'(R)\,\mathcal{B}''(R) - 4\,\mathcal{B}'(R)\right\},$$

$$\tau(R) = \frac{1}{\kappa R\mathcal{A}}\left\{2\,\mathcal{B}'\mathcal{A} - 4R\,\mathcal{A}'\mathcal{B}' + 2\,\mathcal{A}' + 2R^2\,\mathcal{A}'\mathcal{B}'^2 - R\mathcal{A}\mathcal{B}'^2\right\},$$

$$p(R) = \kappa T^3{}_3 = \frac{1-R\mathcal{B}'^2}{\kappa R\mathcal{A}^2}\left\{2R^2\,\mathcal{A}'^2\mathcal{B}' - \mathcal{A}[R^2\mathcal{A}'\mathcal{B}']' + \mathcal{A}^2[R\mathcal{B}']' + \mathcal{A}[R\mathcal{A}']' - 2R\mathcal{A}''\right\}, \quad (20)$$

where

$$\rho(R) = T^0{}_0, \qquad \tau(R) = T^1{}_1, \qquad p(R) = T^2{}_2 = T^3{}_3,$$

with $\rho(R)$ being the *energy density*, $\tau(R)$ is the *radial pressure* and $p(R)$ is *the tangential pressure*. (Note that $\tau(R)$ as defined above is simply the radial pressure p_r, and differs by a minus sign from the conventions in [2, 5].)

Eqs. (20) can be solved to take the form

$$\mathcal{A}(\mathcal{R}) = \frac{1}{k_1 + k_2\sqrt{1-\dfrac{2m}{R}}}, \qquad \mathcal{B}(\mathcal{R}) = \ln\left\{R\left(R - m + R\sqrt{1-\frac{2m}{R}}\right)\right\} - 2\sqrt{1-\frac{2m}{R}}, \qquad (21)$$

where k_1 and k_2 are constants of integration. The associated Riemannian metric of solution (21) takes the form

$$ds^2 = -\eta_1(R)dT^2 + \frac{dR^2}{\eta_2(R)} + R^2d\Omega^2, \quad where \quad \eta_1(R) = \left(k_1 + k_2\sqrt{1-\frac{2m}{R}}\right)^2, \quad \eta_2(R) = \left(1 - \frac{2m}{R}\right),$$

$$(22)$$

The Second Solution

If the unknown function $\mathcal{B}(\mathcal{R}) = 1$, then the resulting partial non linear differential equations have the form

$$\rho(R) = \frac{\mathcal{D}}{\kappa} \left\{ 3\mathcal{D} + 2R\mathcal{D}' \right\},$$

$$\tau(R) = -\frac{\mathcal{D}(1 - R\mathcal{D}'^2)}{\mathcal{A}^2\kappa} \left\{ 3\mathcal{D} + 2R\mathcal{D}' \right\},$$

$$p(R) = \kappa T^3{}_3 = \frac{1}{\kappa R \mathcal{A}^2} \left\{ R^3 \mathcal{A}^2 \mathcal{D}\mathcal{D}'' + 6R^2 \mathcal{A}^2 \mathcal{D}\mathcal{D}' + 3R\mathcal{A}^2\mathcal{D}^2 + \mathcal{A}\mathcal{A}' - 4R^2 \mathcal{A}\mathcal{D}^2\mathcal{A}' \right.$$

$$\left. - (2R\mathcal{A}'^2 - R\mathcal{A}\mathcal{A}'')(1 - R^2\mathcal{D}^2) - 3R^3 \mathcal{A}\mathcal{D}\mathcal{A}'\mathcal{D}' + R^3 \mathcal{A}^2 \mathcal{D}'^2 \right\}. \tag{23}$$

Eqs. (23) can be solved to take the form

$$\mathcal{A}(\mathcal{R}) = \frac{1}{k_2 + \dfrac{k_1}{\sqrt{1 - \dfrac{2m}{R}}}}, \qquad \mathcal{D}(\mathcal{R}) = \sqrt{\frac{2m}{R^3}}. \tag{24}$$

Using (21) or (24) in (11) we can get the components of the energy-momentum tensor turn out to have the form

$$\rho(R) = 0, \qquad \tau(R) = -\frac{1}{\kappa}\left[\frac{2mk_1}{R^3\left(k_1 + k_2\sqrt{1 - \dfrac{2m}{R}}\right)} \right], \qquad p(R) = \frac{1}{\kappa}\left[\frac{mk_1}{R^3\left(k_1 + k_2\sqrt{1 - \dfrac{2m}{R}}\right)} \right]. \tag{25}$$

The weak energy

$$\rho \geq 0, \qquad \rho + \tau \geq 0, \qquad \rho + p \geq 0, \tag{26}$$

and null energy conditions

$$\rho + \tau \geq 0, \qquad \rho + p \geq 0 \tag{27}$$

are both violated as is clear from (25). The violation of the energy condition stems from the violation of the inequality $\rho + \tau \geq 0$.

The associated Riemannian space of solution (24) has the form (22). If one replacing k_2 by $-k_2$ at the above solutions, ((21) or (24)), the resulting form will also be a solution to the non linear partial differential equations ((20) and (23)). The Ricci scalar tensor vanishing, i.e.,

$$R(\{\}) = 0,$$

for solutions (21) and (24). The metric (22) makes sense only for $R \geq 2m$ so to really make the wormhole explicit one needs two conditions patches

$$R_1 \in (2m, \infty), \qquad R_2 \in (2m, \infty),$$

which we then have to sew together at $R = 2m$.

Transforming the metric (22) to the isotropic coordinate using the transformation

$$R \rightarrow \bar{R}\left(1 + \frac{m}{2\bar{R}}\right)^2. \tag{28}$$

Using (28) in (22), then the transformed metric will have the form

$$ds^2 = -\eta_1(\bar{R})dT^2 + \eta_2(\bar{R})\left[d\bar{R}^2 + \bar{R}^2 d\Omega^2\right], \tag{29}$$

where $\eta_1(\bar{R})$ and $\eta_2(\bar{R})$ have the form

$$\eta_1(\bar{R}) = \left(k_1 + k_2 \left[\frac{1 - \frac{m}{2\bar{R}}}{1 + \frac{m}{2\bar{R}}}\right]\right)^2, \qquad \eta_2(\bar{R}) = \left(1 + \frac{m}{2\bar{R}}\right)^4, \tag{30}$$

and the energy momentum tensor given by Eq. (25) takes the form

$$\tau(R) = -\frac{128k_1m}{\kappa}\left[\frac{1}{\bar{R}^2\left(1 + \frac{m}{2\bar{R}}\right)^5\left[2\bar{R}(k_1 + k_2) + m(k_1 - k_2)\right]^2}\right],$$

$$p(R) = \frac{64k_1m}{\kappa}\left[\frac{1}{\bar{R}^2\left(1 + \frac{m}{2\bar{R}}\right)^5\left[2\bar{R}(k_1 + k_2) + m(k_1 - k_2)\right]^2}\right]. \tag{31}$$

As is clear from Eqs. (31) that the energy momentum tensor $\tau(R)$ and $p(R)$ have a common singularity, i.e., $\bar{R} \approx 0$. This is due to the advantage of the isotropic coordinates. When the geometry is such that it can be interpreted as a Lorentzian wormhole then the isotropic coordinate patch is a global coordinate patch. Now one has a single coordinate patch for the traverse wormhole which will be use to discuss the properties of the geometry.

Let us explain this result by studying the geometry of each solution.
1) The geometry of the two solutions (21) and (24) is invariant under simultaneous sign flip

$$k_2 \rightarrow -k_2, \qquad k_1 \rightarrow -k_1,$$

it is also invariant under simultaneous inversion

$$\bar{R} \rightarrow \frac{m^2}{4\bar{R}} \qquad and \;\; sign \;\; reversal \qquad k_2 \rightarrow -k_2,$$

keeping k_1 unchanged.

2)
$$k_2 \neq 0, \qquad k_1 = 0,$$
gives the Schwarzschild geometry, it is non-traversable.

3)
$$k_2 = 0, \qquad k_1 \neq 0,$$

4)
$$k_2 = 0, \qquad k_1 = 0$$

is a singular.

5) At the throat $g_{tt}(r = 2m) = -k_1^2$, so $k_1 \neq 0$ is required to ensure traversability.

6) To see if there is ever horizon or not Dadhich et. al. [13] done the following discussion.

A horizon would seem to form if the component $g_{tt} = 0$, i.e., if there is a physically valid solution to

$$k_1(1 + \frac{m}{2\bar{R}}) + k_2(1 - \frac{m}{2\bar{R}}) = 0, \tag{32}$$

whose solution has the form

$$\bar{R}_h = \frac{m}{2}\frac{k_2 - k_1}{k_2 + k_1}, \tag{33}$$

that is a horizon tries to form if

$$\frac{k_2 - k_1}{k_2 + k_1} > 0, \tag{34}$$

which is actually a naked singularity. This singularity occurs if either

$$k_2 + k_1 > 0, \qquad and \qquad k_2 - k_1 > 0,$$
$$or$$
$$k_2 + k_1 < 0, \qquad and \qquad k_2 - k_1 < 0. \tag{35}$$

Outside of these regions the naked singularity does not form and one has a traversable wormhole. Dadhich et. al. [13] studied the $k_1 - k_2$ plan and have found that if

$$k_2 = Z \cos\theta, \qquad k_1 = Z \sin\theta,$$

i) $\theta = 0$: Schwarzschild spacetime.

ii) $\theta \in (0, \pi/4)$, naked singularity.

iii) $\theta = \pi/4 \Rightarrow k_2 = k_1$, i.e., the $\bar{R} \to 0$ region is not flat.

iv) $\theta \in (\pi/4, 3\pi/4)$ traversable wormhole.

v) $\theta = \pi/2$ is the spatial-Schwarzschild wormhole.

vi) $\theta = 3\pi/4 \Rightarrow k_2 = -k_1$, the $\bar{R} \to \infty$ region is not flat.

vii) $\theta \in (3\pi/4, \pi)$ naked singularity.

viii) $\theta = \pi$: Schwarzschild spacetime.

ix) $\theta > \pi$: repeat the previous treatment.

The ratio value of the two parameters k_1 and k_2 that characterize the solutions (21), (24) is obtained $k_1/k_2 \geq 10^8\gamma$ with $\gamma^2\beta^2 < 2$ implying $\beta < \sqrt{2/3}$ [13] .

Thus we have two exact solutions of the field equations (11), each of which leads to the same metric as given by Eq. (22). The axial vector part of the two solutions (21) and (24) vanishing identically i.e., $a_\mu = 0$. Therefore, the two tensors $H_{\mu\nu}$ and $K_{\mu\nu}$ are also vanishing and Møller's tetrad theory coincides in that case with the teleparallel equivalent of general relativity.

4. Energy content

The superpotential is given by

$$\mathcal{U}_\mu{}^{\nu\lambda} = \frac{(-g)^{1/2}}{2\kappa} P_{\chi\rho\sigma}{}^{\tau\nu\lambda} \left[\phi^\rho g^{\sigma\chi} g_{\mu\tau} - \lambda g_{\tau\mu} \gamma^{\chi\rho\sigma} - (1-2\lambda) g_{\tau\mu} \gamma^{\sigma\rho\chi} \right], \tag{36}$$

where $P_{\chi\rho\sigma}{}^{\tau\nu\lambda}$ is

$$P_{\chi\rho\sigma}{}^{\tau\nu\lambda} \stackrel{\text{def.}}{=} \delta_\chi{}^\tau g_{\rho\sigma}{}^{\nu\lambda} + \delta_\rho{}^\tau g_{\sigma\chi}{}^{\nu\lambda} - \delta_\sigma{}^\tau g_{\chi\rho}{}^{\nu\lambda} \tag{37}$$

with $g_{\rho\sigma}{}^{\nu\lambda}$ being a tensor defined by

$$g_{\rho\sigma}{}^{\nu\lambda} \stackrel{\text{def.}}{=} \delta_\rho{}^\nu \delta_\sigma{}^\lambda - \delta_\sigma{}^\nu \delta_\rho{}^\lambda. \tag{38}$$

The energy-momentum density $\tau_\mu{}^\nu$ is defined by

$$\tau_\mu{}^\nu \stackrel{\text{def.}}{=} \mathcal{U}_\mu{}^{\nu\lambda}{}_{,\,\lambda},$$

and automatically satisfies the conservation law, $\tau_\mu{}^\nu{}_{,\,\nu} = 0$. The energy is expressed by the surface integral [33, 34, 35]

$$E = \lim_{r\to\infty} \int_{r=constant} \mathcal{U}_0{}^{0\alpha} n_\alpha dS, \tag{39}$$

where n_α is the unit 3-vector normal to the surface element dS.

Now we are in a position to calculate the energy associated with solution (21) using the superpotential (36). As is clear from (39), the only components which contributes to the energy is $\mathcal{U}_0{}^{0\alpha}$. Thus substituting from solution (21) into (36) we obtain the following non-vanishing value

$$\mathcal{U}_0{}^{0\alpha} = \frac{2mn^\alpha}{\kappa R^2}. \tag{40}$$

Substituting from (40) into (39) we get

$$E = m. \tag{41}$$

Repeat the same calculations for solution (24) we obtain the necessary components of the superpotential

$$\mathcal{U}_0{}^{0\alpha} = \frac{4mn^\alpha}{\kappa R^2}, \tag{42}$$

and the energy will have the form

$$E = 2m. \tag{43}$$

5. Discussion and conculusion

In this paper we have applied the tetrad having spherical symmetry with three unknown functions of radial coordinate [29] to the field equations of Møller's tetrad theory of gravitation [15]. From the resulting partial differential equations we have obtained two exact non vacuum solutions. The solutions in general are characterize by three parameters m, k_1 and k_2. If the two parameters $k_1 = 0$ and $k_2 = 1$ then one can obtains the previous solutions when the exponential term equal zero, i.e., $e^{-R^3/r1^3} = 0$ [36]. The energy-momentum tensor has the property that $\rho(R) = 0$ for the two solutions. This leads to the violation of the weak energy and the null energy conditions defined by Eqs. (26) and (27) due to the fact that the radial pressure is negative, i.e., $\tau(R)$ as is clear from Eq. (25). The line element associated with these solutions has the same form (22).

To make the picture more clear we discuss the geometry of each solution. The line element of these solutions in the isotropic form are given by (29). If $g_{tt} = 0$ one obtains a real naked singularity region. Outside these regions naked singularity does not form and one obtains a traverse wormhole. The throat of this wormhole $g_{tt}(R = 2m)$ gives the conditions that $g_{tt} = -k_1{}^2 \Rightarrow (k_1 \neq 0$ is required to ensure the traversability). The properties of this wormhole are discussed by Dadhich et. al. [13].

It was shown by Møller [16, 15] that the tetrad description of the gravitational field allows a more satisfactory treatment of the energy-momentum complex than does general relativity. We have then applied the superpotential method [16, 15] to calculate the energy of the gravitating system of the two solutions (21) and (24). As for the first solution we obtain $E = m$ and there is no effect of the two parameters k_1 and k_2 characterize the wormhole. As for the second solution the energy is $E = 2m$ and this is due to the fact that the time-space components of the tetrad fields $e_0{}^\alpha$, $e_\alpha{}^0$ go to zero as $\dfrac{1}{\sqrt{r}}$ at infinity [35, 36]. The disappearance of the two parameters k_1 and k_2 from the two Eqs. (41) and (43) may give an impression that these two parameters are not a physical quantities like the gravitational mass m.

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